AN OSCILLATOR MODEL FOR ANALYSIS OF GUITAR SOUND PRESSURE RESPONSE

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Abstract

We investigate sound pressure response of classical guitars in the region of frequencies up to 6-800 Hz. In this range, the response spectrum is characterized by resonance peaks corresponding to vibrational modes of the top plate. We model guitar response as a superposition of contributions from single resonances. Each resonance is modelled as a harmonic oscillator, moving a piston and acting as a simple monopole radiator. We find that this simple model adequately describes guitar responses up to 6-800 Hz. Theoretical fits to response curves make it possible to determine for each resonance (oscillator) the ratio A/m (piston area to oscillator mass). The net sound radiated from the oscillator is proportional to this ratio. Data for five good classical guitars are presented. The implication of this work is, that guitar responses up to 6-800 Hz can be characterized by three parameters for four to six resonances instead of by raw data points.

Introduction

The sound radiated from the guitar is mainly generated by the vibrating top plate. One way to characterize a guitar is by measuring the sound pressure response for a sinusoidal constant-force excitation, usually applied to the bridge. The sound pressure response shows well-defined resonance peaks at frequencies from approx. 100 Hz up to about 6-800 Hz, depending on the individual instrument. Hologram-interferometric studies of the top plate have shown (Jansson, 1971) that the resonances correspond to characteristic modes of vibration of the top plate. For the lower resonances, the top plate vibrates in modes with few nodal lines as shown in Fig. 1. At higher frequencies, the sound pressure response from many overlapping resonances forms a 'resonance continuum' (Caldersmith, 1981) with a multitude of peaks and antiresonances. At these frequencies, the top plate vibrates in increasingly smaller subdivisions.

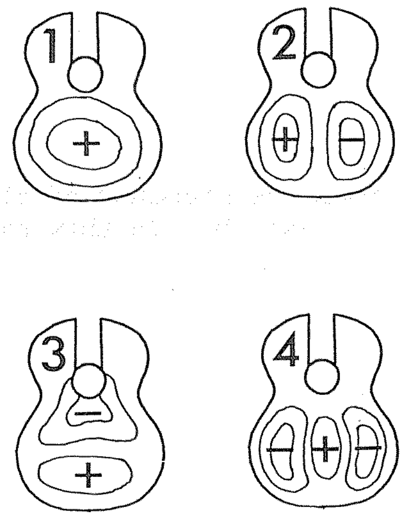

Fig. 1. The vibrational configurations of the four lowest top plate modes found in most classical guitars. Typical resonance frequencies are 200, 300, 400, and 500-550 Hz. Contours of the same vibrational amplitude are indicated. The relative direction of vibration (up/down) is indicated with plus and minus. The second resonance in the figure is a pure dipole. The third and fourth resonances contain both monopole and multipole radiation.

The purpose of this paper is to give an understanding of guitar sound pressure response in the region of frequencies up to 6-800 Hz where the response is characterized by resonance peaks.

We shall assume here, that each vibratory mode can be represented as a simple harmonic oscillator. The lowest top-plate mode is a pure monopole source of radiation. Modes with more nodal lines in general produce both multipole and net monopole radiation. Since multipole radiation is inefficient at lower frequencies, we only consider the monopole part of the radiation from each oscillator.

The first top-plate is a pure monopole source and is coupled to the Helmholtz resonance of the air cavity. We have earlier (Christensen and Vistisen, 1980) given a quantitative description of this system. In this paper, we focus on the response over a broader frequency range and sacrifice the more detailed description of the two lowest resonances. Thus, the lowest resonance at around 100 Hz is omitted and the second resonance at around 200 Hz is treated as if it is due entirely to the first top-plate resonance.

We present a simplified model of guitar response at frequencies up to 6-800 Hz. The theoretical concepts employed are the harmonic oscillator combined with the sound pressure response from a simple monopole source of acoustical radiation. We explore to which extent the frequency response from a classic guitar can be described by a superposition of responses from harmonic oscillators, each of which acts as a simple source of monopole radiation. Each oscillator is characterized by its resonance frequency and Q-factor together with the ratio of its effective piston area to effective mass as seen from the driving point.

Theory

The harmonic oscillator

The basic theoretical element in this paper is the simple harmonic oscillator which moves a piston and, hence, gives rise to acoustic monopole radiation. The oscillator might be thought of as a loudspeaker enclosed in a cabinet. Let x denote the distance of the piston from its equilibrium position. The oscillator, of mass m and with stiffness constant k, is acted upon by a force F according to Newtons second law:

$$m\ddot{x} = F-kx-R\dot{x}$$

where R is the resistance to motion. For a sinusoidally varying force, the motion is also sinusoidal and the above equation can be solved for the oscillator velocity u

$$u = \frac{F}{m} \frac{i\omega}{(\omega_0^2 - \omega^2) + i\gamma\omega} \tag{1}$$

Here, the resonance frequency f_o is given by $\omega_o = 2\pi f_o$, $\omega_o^2 = k/m$ and γ equals R/m. In terms of the Q-factor $\gamma = 2\pi f_o/Q$.

The moving piston acts as a source of monopole radiation. At a distance r from the source, the magnitude of sound pressure is given by

$$p = -\frac{i\omega\rho}{4\pi r} uA \tag{2}$$

where ρ is the density of air (1.205 kg/m^3). The variation of phase with distance from the source is not important for the present purpose. Using the piston velocity from Eq. (1) we obtain for the sound pressure:

$$p = F \frac{A}{m} \frac{\rho}{4\pi r} \frac{\omega^2}{(\omega_0^2 - \omega^2) + i\gamma\omega} \tag{3}$$

At a given distance, the sound pressure is proportional to the ratio of piston area to mass A/m. The last factor accounts for the frequency variation. The pressure is positive for $f \ll f_o$ and negative for $f \gg f_o$. For low frequencies, the magnitude of the sound pressure is proportional to f^2 whereas at high frequencies, the response becomes constant, proportional to A/m.

The real guitar

The guitar is a vibratory system characterized by many resonances. The lowest resonances typical for most classic guitars are shown in Fig. 1. In the measurements presented here, the guitar was excited by a constant force transducer at the center of the bridge. Each of the resonances may be characterized by an effective piston area and an effective mass. For the more complicated vibrational configurations as, for instance, modes no. 3 and 4 in Fig. 1, some parts of the top plate move 180 degrees out of phase with the point of excitation. In such cases, the effective monopole piston area is defined as the area which, when moving with the velocity of the point of excitation, produces the actual net volume displacement of the source. Mathematically speaking, this relation may be formulated as:

$$A_i\, u_{exc} = \int_{\text{guitar face}} u_i(x,y)\, da \tag{4}$$

where $u_i(x,y)$ is the velocity of the point (x,y) of the guitar top plate for the i'th resonance and where u_{exc} is the velocity at the point of excitation.

It follows that the effective piston area can be negative, i.e., that the net volume displacement takes place at a phase opposite to that of the point of excitation. Indeed we shall show that the great variability in guitar response curves for different guitars is due to various combinations of positive and negative piston areas.

The effective mass of a particular mode depends upon the position of the exciter. If the exciter is placed close to a nodal line, the effective mass of that mode becomes large. The ratio A/m may therefore change drastically when the point of excitation is changed.

The sound pressure from the i'th resonance is a function of A_i/m_i, f_{oi}, and Q_i, i.e., $p = p(f, A_i/m_i, f_{oi}, Q_i)$. Therefore the total sound pressure from the guitar - not counting multipole radiation - is given as

$$P_{tot}(f) = \sum_i p(f, A_i/m_i, f_{oi}, Q_i) \tag{5}$$

Since the contribution from one oscillator grows as f^2 and reaches a constant level above resonance, it follows that at any frequency f, the sound pressure is mainly determined by oscillators for which $f_{oi} \leq f$. In

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fitting a series of oscillators to describe a measured sound pressure response, one can start by fitting the first oscillator, then the second oscillator etc. because the contribution from oscillators at higher frequencies is marginal due to the f^2-dependence of response below resonance.

Theoretical examples

In the following we will give some theoretical examples on sound pressure response curves resulting from the superposition of two and three oscillators. The first resonance, at the lowest frequency, represents the first top plate mode of the guitar which corresponds to the second resonance of the guitar. The ratio of the effective top plate area to mass has been chosen to be largest for this resonance, in accordance with the experimental findings presented later.

The two-oscillator case

The contribution to sound pressure from a resonance is positive below resonance and negative above resonance. Thus, for two resonances with the same sign of the piston area, the contributions of the oscillators tend to cancel each other between resonances, leading to an antiresonance between the two resonances, as shown in Fig. 2. The antiresonance occurs close to the cross-over frequency of the individual response curves. Above the highest resonance the two modes vibrate in phase and, therefore, reinforce each other.

If the two oscillators have the opposite sign of the piston area, the response is increased between the resonances since the phase of both oscillators is the same here. Above the highest resonance, the contributions from the two oscillators tend to make them cancel each other and an antiresonance occurs close to the cross-over frequency of the individual response curves. If, however, the two oscillators have opposite sign of

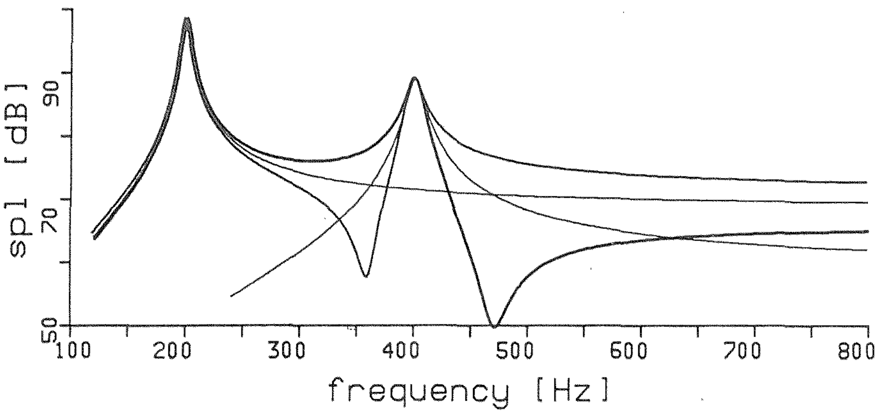

Fig. 2. Sound pressure response from two superposed oscillators. The oscillators have resonance frequencies of 200 and 400 Hz and A/m values of 6 and 2 cm^2/g. The Q-factors of both oscillators are 30. The tine lines give the contributions of the individual oscillators. The medium-heavy line (antiresonance between resonances) shows the response when both piston areas are positive. The heavy line (antiresonance after second resonance) shows the response when the second piston area is negative. The sound pressure is calculated at a distanceof 1 m with an exciting force of 1 N.

piston area and if the A/m-ratio of the first one is sufficiently small, there is no antiresonance after the second resonance. Such a case is seen if one tries to fit the two first resonances of a guitar, i.e., the coupled Helmholtz and first top plate resonances.

The three-oscillator case

In Fig. 3 the sound pressure response is shown for a system of three oscillators. There are four possible nonidentical combinations of signs of the piston areas. The individual curves of Fig. 3 can all be understood from the previous two-oscillator case. The piston area of the first resonance at 200 Hz is chosen positive. The structure around the second resonance at 400 Hz is understood from the position of the antiresonance which occurs before the peak if the piston area is positive and after the peak if the piston area is negative. At 600 Hz, the combined

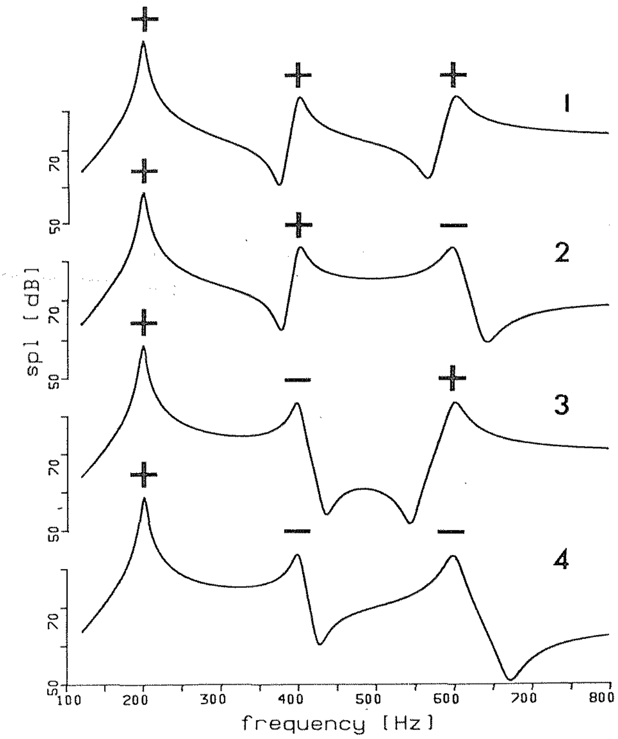

Fig. 3. Sound pressure response from three superposed oscillators. The oscillators have resonance frequencies at 200, 400, and 600 Hz. The Q-factors of all oscillators are 30 and the A/m-ratios of the oscillators are 6, 1 and 1 cm^2/g. The four response curves represent situations with different combinations of the signs of the three piston areas, as indicated above each resonance peak. The sound pressure is calculated at a distance of 1 m with an exciting force of 1 N.

two first resonances act roughly as one piston, since the response of an oscillator approaches a constant level at frequencies well above resonance. The structure around the third resonance is, thus, again explained by the position of the antiresonance.

The situation represented by the top curve in Fig. 3 has - to the author's knowledge - never been seen in a guitar. The situation in the second curve from the top is found in guitars with a pronounced second resonance, cf. Fig. 1. The three peaks in the curve represent then the first, second, and third top plate modes. The third curve from the top represents a rather undesirable situation in which two antiresonances fall in-between two neighbouring resonances and create a region of poor acoustical response. The bottom curve shows a situation found in guitars with a second top-plate resonance which not is excited when the exciter is positioned at the center nodal line of this mode, see Fig. 1. The resonances in this curve correspond to the first, third, and fourth top plate modes in most classic guitars.

Thus, many qualitative features of guitar response curves may be understood from this simple model of superimposed harmonic oscillators. The variability of response curves is brought about by the combinations of different signs of piston area of the individual resonances.

Experimental details

Measurements

The sound pressure response curves for the five guitars studied here were measured as described earlier (Christensen and Vistisen, 1980) in an anechoic chamber. The sound pressure level was measured 2 m above the guitar top plate. The exciting force of approximately 0.2 N was applied to the center of the bridge. Response curves for the five guitars are shown in Fig. 4. In order to facilitate comparisons with theoretical calculations, the sound pressure response curves were scaled to represent values at 1 m distance from the guitar for an exciting force of 1N.

The instruments

The guitars used in this study were all handcrafted instruments with spruce top plates. All guitars have rosewood back and sides with the exception of no. 3, which has cypress back and sides. Further details of the instruments are listed below.

1) Ramirez: The bracing has a diagonal bar crossing the transverse bar below the soundhole and running down on the treble side of the top plate; serial no. 4.952, Madrid, 1971.

2) Ibanez: This is essentially a Japanese version of a Ramirez guitar; bracing as described on guitar 1.

3) Taurus: Traditional Torres bracing; serial no. 56, Barcelona, 1967.

4) Contreras: Flamenco guitar with traditional Torres bracing; Madrid, ca. 1977.

5) Romanillos: Torres bracing with transverse thin plate about the size of the bridge plated below the bridge; serial no. 224, England, 1978.

Results and discussion

Theoretical response curves from superposed harmonic oscillators were fitted to experimental response curves as shown in Fig. 4. The initial data used in theoretical calculations were the resonance frequencies from the experimental response curves together with tentative estimates of their Q-factors. The signs on the piston areas of the resonance peaks were initially chosen from a judgement of the position of the antiresonances as described in the preceding chapter. The magnitudes of the A/m-ratio for the resonances and the final Q-factors were adjusted to obtain the visually best fit of the calculated response to the experimental one as judged from a plot of these curves.

The calculated response curves are shown in Fig. 4 together with the measured responses. The data used in the calculations are shown in Table 1.

In general, there is a good agreement between calculated response curves and measured ones over a large part of the frequency region studied. The oscillator model explains the structure of the response curves and even provides a good quantitative agreement. No fit was attempted

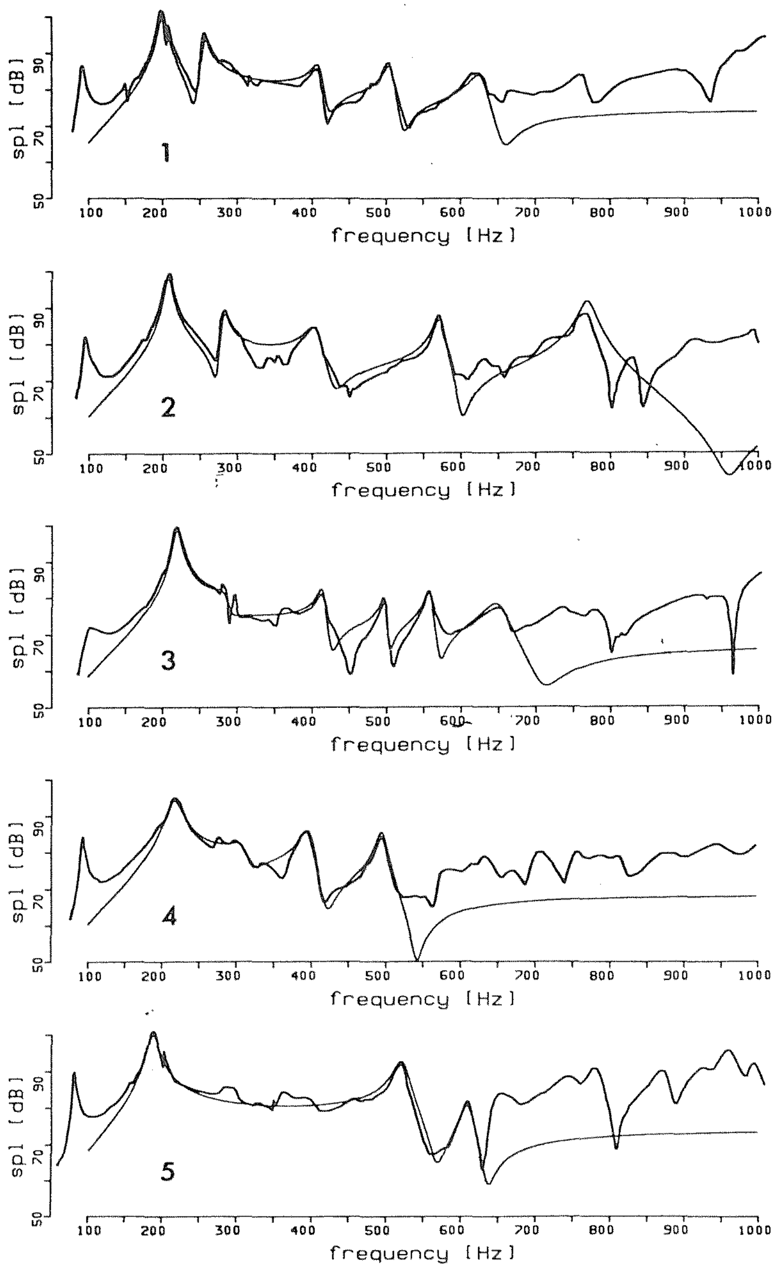

Fig. 4. Comparison between measured sound pressure responses and calculated ones for five guitars. Thin lines are calculated responses and heavy lines are experimental ones. All curves are scaled to apply for an exciting force of 1 N at a distance of 1 m from the guitar top plate. The oscillator parameters used in the calculation are shown in Table 1.

Table 1. Oscillator parameters used in calculating the theoretical response curves in Fig. 4.

Guitar	f_0 (Hz)	Q	A/m (cm^2/g)	top-plate mode
1:Ramirez	200	25	10.	1
-	257	25	4.	2
-	410	30	-1.2	3
-	506	40	-1.0	4
-	627	30	-1.0	?
2:Ibanez	208	25	7.	1
-	282	35	1.5	2
-	405	20	-1.5	3
-	572	50	-1.0	4
-	770	40	-2.0	?
3:Taurus	220	25	7.5	1
-	285	15	-0.8	2
-	415	40	-0.6	3
-	498	70	-0.25	4
-	559	60	-0.4	?
-	650	20	-0.8	?
4:Contreras	216	12	9.0	1
-	310	15	-1.	2
-	395	25	-1.5	3
-	495	40	-1.0	4
5:Romanillos	187	15	14.0	1
-	522	30	-3.0	4
-	610	50	-0.5	?

below 150 Hz because the structure of the two first resonances - at approximately 100 and 200 Hz - has already been explained quantitatively as a result of a coupling between the Helmholtz and first top plate resonances (Christensen and Vistisen, 1980). Only the highest of these resonances is taken into account because the aim of this work is to test if response curves may be fitted to the superposed oscillator model rather than to give a detailed account of the nature of each resonance.

Above 600-800 Hz it was not possible to fit the sound pressure response by superposed oscillators. At these 'high' frequencies there is no structure characteristic of resonances. It is known from hologram-interferometric studies (Jansson, 1971; Firth, 1977) that resonances at high frequencies still may be characterized by simple geometric patterns, as the ones in Fig. 1, but with an increasing number of nodal lines on the guitar top plate. The net monopole radiation from such resonances decreases while at the same time multipole radiation becomes more efficient. Caldersmith (1981) has characterized this region of guitar response as a 'resonance continuum' with a strong directional dependence of the radiated sound.

In contrast, the region of frequencies studied here (up to 6-800 Hz) is characterized by strong sources of net monopole radiation - air 'pumping' modes - with little directional dependence.

Table 1 gives a list of the parameters used in the fitting to the measured response curves. Each resonance is given a tentative assignment to a corresponding top plate mode. Such an assignment is based on the author's investigation of the mode structure at resonance of many guitars. No such specific assignment was undertaken of these instruments. For the low-order top plate modes assigned here, there is little doubt of the correctness of the assignment which follows the one observed in a number of hologram-interferometric studies (Jansson, 1971; Firth, 1977; Schwab, 1975).

Comments to Table 1

The first top-plate mode at around 200 Hz has piston area to mass ratios ranging from 7 to 14 cm^2/g. These values are slightly higher than the ones found from an analysis of the two lowest resonances (Christensen and Vistisen, 1980) because we have not accounted for the Helmholtz resonance. The A/m-ratio for the first top-plate mode is almost one order of magnitude larger than for the higher resonances. The contribution to sound pressure from one oscillator approaches a constant at high frequencies. Therefore, the magnitude of the A/m-ratio for the first top-plate mode is important for the behaviour at high frequencies too. If $(A/m)_1$ is reduced by a factor of two, the sound pressure level is reduced by about 3 dB between the higher resonances. The sound pressure level between the resonances is rather important. The partials of a tone which fall in-between the resonances have a longer sustain, because the energy of the vibrating string is drained very fast in the vicinity of the resonances.

The second top-plate mode at 260-310 Hz is usually a pure dipole in guitars with a symmetrical bracing (see Fig. 1) but it can be turned into a strong monopole source if the bracing is nonsymmetrical. Guitars no. 3 and 4 have symmetrical bracing and, accordingly, we find that the second top-plate mode is characterized by a rather small negative A/m-ratio and a poor Q-factor, probably because this mode is excited very close to the center nodal line. As seen from Fig. 4, this mode is rather insignificant and for guitars no. 3 and 4 it has only been accounted for because it gives a small 'cosmetic' improvement of the fit to the measured response. On the contrary, guitars no. 1 and 2 with nonsymmetrical bracing show a strong monopole contribution from the second top-plate mode with a relatively large positive A/m-ratio and a fairly high Q-factor.

The third top-plate mode occurs close to 400 Hz and is probably coupled to the half-wave longitudinal resonance in the air cavity. For all instruments studied, this resonance had a negative A/m-ratio. This is also the case for all of the higher-order resonances. A probable reason for this is, that for the higher-order modes, most of the motion takes place at the outer lobes of the top plate because the center is made stiff by the presence of the bridge.

The fourth top-plate resonance can be identified in all guitars studied. It occurs at 500 to 570 Hz. For all but one guitar, it was possible to identify at least one additional resonance at higher frequencies before the sound pressure response approaches a resonance continuum with no characterstic resonance peaks.

In conclusion, we find that the first four top-plate resonances account for the sound pressure response up to about 600 Hz. In addition, there might be higher air-pumping resonances up to about 800 Hz. At still higher frequencies - in the 'resonance continuum' - it is not possible to fit guitar sound pressures by the present model.

Conclusion

The purpose of this paper was to explain the sound pressure response curves for the classical guitar. This aim has been reached to the extent that we now have a qualitative understanding of response curves. The simple principles outlined in the sections on 'the two-oscillator case' and 'the three-oscillator case' show that the behavior between resonances can be explained from an understanding of the harmonic oscillator piston. The very different behavior obtained between the resonances (see Figs. 2 and 3) is due to the different combinations of the signs and the magnitudes of piston areas for different vibrational modes and are, hence, related to the vibrational structure of the top-plate modes.

Qualitatively we have shown that guitar response curves may be accounted for up to 6-800 Hz by superimposing responses from harmonic oscillators, each of which acts as a simple source of monopole radiation. Each oscillator corresponds to a resonance peak in the response curve. The correspondence obtained between measured response curves and model calculations implies that the response up to 6-800 Hz is dominated by monopole radiation, mainly contributed by the first four top-plate resonances.

Recently we have found (Christensen, 1983) that most of the acoustical energy present in long-time-average spectra of played classical guitar music originates from the frequency region 200 to 800 Hz. It follows that the region, which contributes mostly to the radiated acoustical energy, is dominated by radiation from monopole sources. Further progress in guitar making may, thus, be achieved by devicing practical methods to tune the top plate to provide good responses at the first four top plate modes.

Understanding sound pressure response curves have some implications:

It is much easier to characterize a guitar by the parameters of some four to six harmonic oscillators than by the frequency response curves. This is particularly useful in comparing different instruments because the subjective impression of quality may be correlated to oscillator parameters. One can in this way gain an understanding of the physical characteristics that are desirable from a subjective impression of instrument quality.

It is interesting to note that in principle the mode frequencies and vibration amplitudes of a given top plate may be computed theoretically (Schwab, 1976). Except for Q-factors, such a computation could give information about the A/m-ratios and resonance frequencies for a given top-plate design. The present model could then be used to calculate the sound pressure response curve. This would be particularly fruitful if one at the same time had a subjective quality evaluation based on oscillator parameters.

Acknowledgments

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References

Caldersmith, G. (1981): "Physics at the workbench of the luthier", Catgut Acoust.Soc. Newsletter No. 35, 15,

Christensen, O. (1983): "The response of played guitars at middle frequencies", Acustica 52,

Christensen, O. and Vistisen, B.B. (1980): "Simple model for low-frequency guitar function", J.Acoust.Soc.Am. 68, 758.

Firth, I. (1977): "Physics of the guitar at the Helmholtz and the first top-plate resonances", J.Acoust.Soc.Am. 61, 588.

Jansson, E.V. (1971): "A study of acoustical and hologram interferometric measurements of the top plate vibrations of a guitar", Acustica 25, 96.

Schwab, H.L. (1975): "Finite element analysis of a Guitar Soundboard", Catgut Acoust.Soc. Newsletter No. 24, 13.

Schwab, H.L. (1976): "Finite element analysis of a Guitar Soundboard - Part II", Catgut Acoust.Soc. Newsletter No. 25, 13.